PHYS 232 – Assignment #5

Due Friday, Mar. 22 @ 11:00

1. You are investigating the diffusion of dye molecules in a solution, and have measured the dye concentration $C(x)$ at time $t = 2.0$ s as a function of position x as given in the table below. Theory predicts that the concentration should follow:

$$C(x) = \frac{A_0}{(4\pi Dt)^{3/2}} \exp\left[-\frac{x^2}{4Dt}\right]$$

Analyze the data to find A_0 and the diffusion constant D, assuming that the errors in position x and time t are negligible. You should be able to find these parameters from a weighted fit to a straight line (Excel won't be good enough) to an appropriate plot of the data. Your answer should include an explanation of what you are plotting and why and how you arrive at your answers. Submit a plot of the linearized data with the best-fit line. Include meaningful error estimates for the parameters A_0 and D.

$C(x)$ (cm^{-3})	x (cm)
0.9965 ± 0.01	0.1
0.8567 ± 0.01	0.5
0.5341 ± 0.003	1.0
0.2415 ± 0.003	1.5
0.0871 ± 0.003	2.0
0.0206 ± 0.001	2.5
0.0034 ± 0.001	3.0
0.0001 ± 0.0005	3.5

2. Suppose that you make a total of N measurements of the quantity $y_i \pm \sigma_i$ as a function of x_i ($i = 1, 2, \ldots, N$). Suppose also that you expect y to depend linearly on x such that $y = a_0 + b_0 x$. In class we derived expressions for the best values of a_0 and b_0. That is, the values of a_0 and b_0 that minimize the function:

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - a_0 - b_0 x_i}{\sigma_i} \right)^2 . \tag{1}$$

The results were:

$$a_0 = \frac{1}{\Delta} \left[\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} \right], \tag{2a}$$

$$b_0 = \frac{1}{\Delta} \left[\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} \right], \tag{2b}$$

where all of the sums are from $i = 1$ to N and:

$$\Delta \equiv \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2} \right)^2 . \tag{3}$$

Using the propagation of error formula we were also able to determine the uncertainty in best estimates of parameters a_0 and b_0:

$$\sigma_a^2 = \frac{1}{\Delta} \sum \frac{x_i^2}{\sigma_i^2}, \tag{4a}$$

$$\sigma_b^2 = \frac{1}{\Delta} \sum \frac{1}{\sigma_i^2}. \tag{4b}$$

Your task for problem 2:

A student is provided with a radioactive source and a Geiger counter. The student wishes to investigate the $1/r^2$ law governing the count rate by recording the Geiger counter measurements over a fixed period of time at various distances from the radioactive source. She acquires the following data:

index i	Distance from Source r_i (m)	Counts C_i	Error in Counts σ_{C_i}
1	0.20	901	30.0
2	0.25	652	25.5
3	0.30	443	21.0
4	0.35	339	18.4
5	0.40	283	16.8
6	0.45	281	16.8
7	0.50	240	15.5
8	0.60	220	14.8
9	0.75	180	13.4
10	1.00	154	12.4

(a) Plot the counts C as a function of $1/r^2$ to show that the data are approximately linear. That is, we can express the counts as a linear function of one over the square of the distance from the source: $C = a_0 + b_0/r^2$.

(b) Use the expressions on the previous page to find the best-fit values for a_0 and b_0. Here you must take C_i as the y data points and $1/r_i^2$ as the x data points. You may use Python or Excel or whatever to evaluate the sums, but you must clearly write down the numerical values of all the sums needed to calculate the final values of Δ, a_0, and b_0. Once you've determined the a_0 and b_0 values, add the best-fit line to your plot in part (a).

(c) Find the uncertainties σ_a and σ_b in your best-fit values for a_0 and b_0.

(d) Use Python to perform a weighted least-squares fit to C as a function of $1/r^2$. Compare the values of $a_0 \pm \sigma_a$ and $b_0 \pm \sigma_b$ obtained in parts (b) and (c) to those obtained using the Python fit. (Note that you must do a weighted linear fit, so Excel won't be sufficient). Submit a plot of the linear data that includes the best-fit line.